Principio di separazione

1) $\dot{x} = Ax + Bu$

$u = Kx$

(A,B) raggiungibile (stabilizzabile)

2) $\dot{\xi} = A\xi + Bu + G(y - C\xi)$

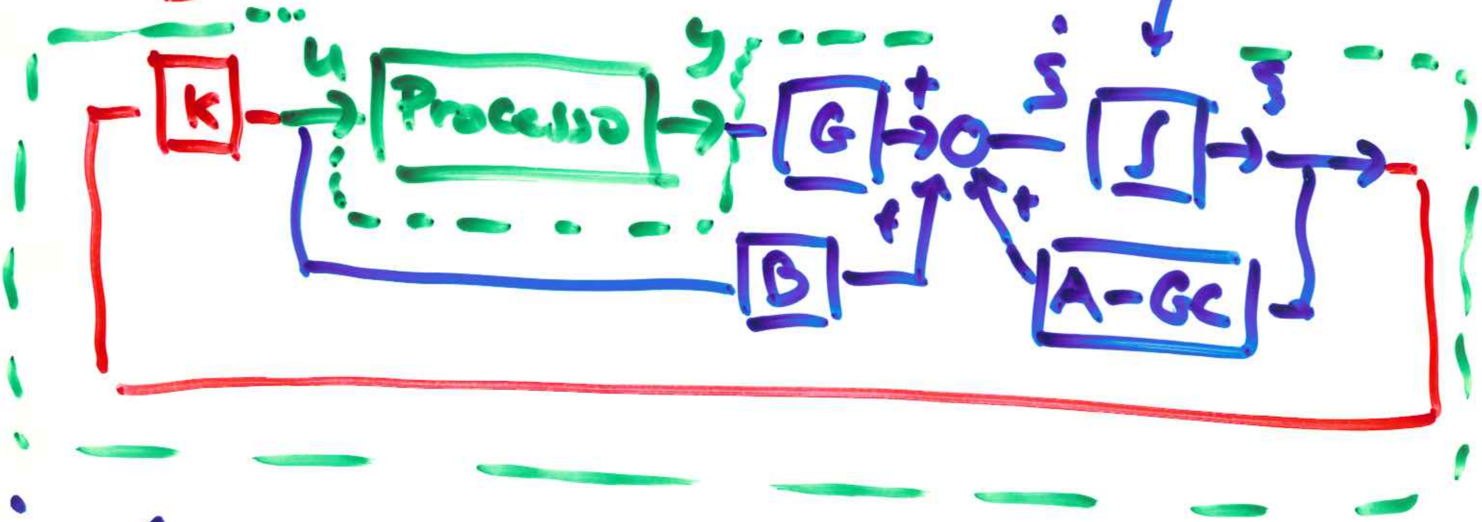
(A,C) osservabile (rilevabile)

$e = x - \xi$

1)+2) ?

n $\int$ in parall.

$u = K\xi$



$\dot{\xi} = (A + BK - GC)\xi + Gy$

$u = K\xi$

$G_c(s) = K\left[sI - (A + BK - GC)\right]^{-1} G = \dfrac{u(s)}{y(s)}$

$$\dot{x} = Ax + Bu = Ax + BK\xi$$

$$\dot{\xi} = (A - GC + BK)\xi + GCx$$

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} * & * \\ * & * \end{bmatrix} \begin{bmatrix} x \\ \xi \end{bmatrix}$$

$$(x, \xi) \to x,\ e = x - \xi \qquad \left(T = \begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}\right)$$

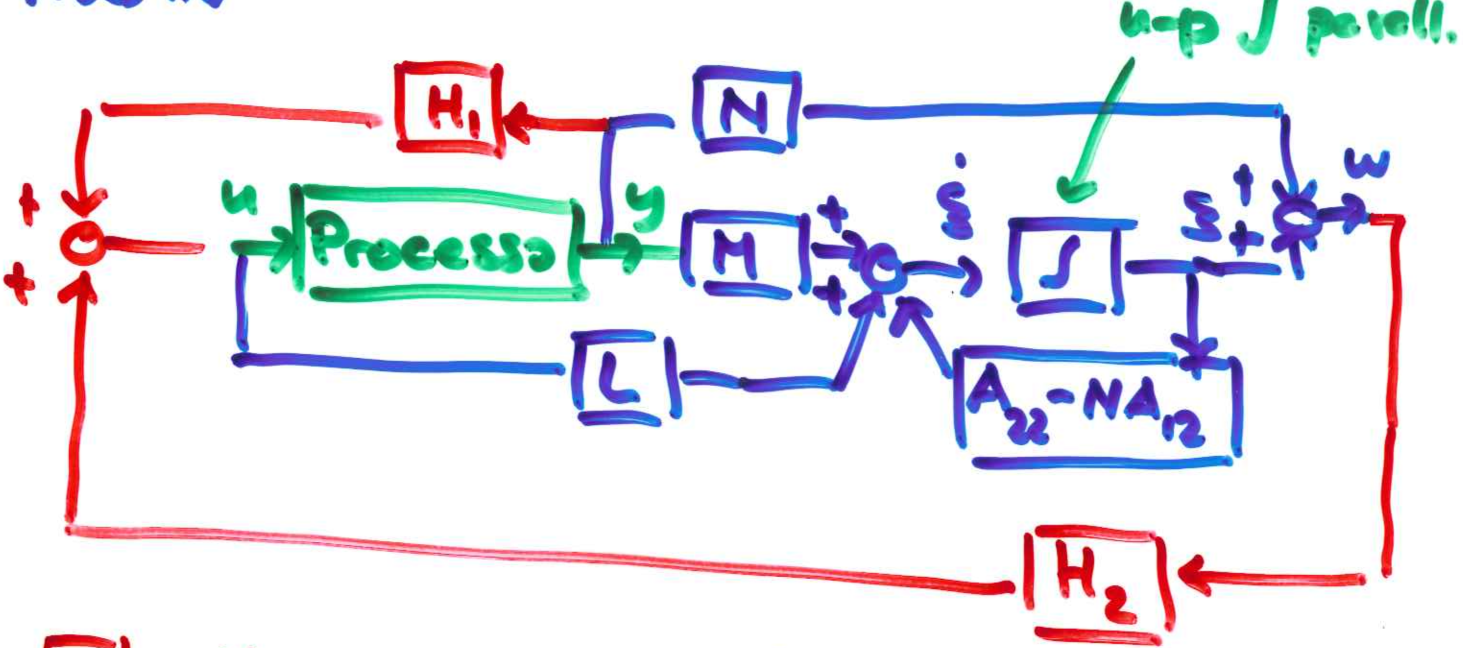
$$\dot{x} = (A + BK)x - BKe$$

$$\dot{e} = * \quad (A - GC)e$$

$$A_{cl} = \begin{bmatrix} A + BK & -BK \\ 0 & A - GC \end{bmatrix}$$

$$\sigma(A_{cl}) = \sigma(A + BK) \cup \sigma(A - GC)$$

Controllore stabilizzante basato sull'osservatore ridotto



$$\boxed{u} = H_1 y + H_2 w = \underbrace{(H_1 + H_2 N) y + H_2 \xi}$$

$$\dot{\xi} = (A_{22} - N A_{12}) \xi + M y + L u$$

$$\Downarrow$$

$$\dot{\xi} = F \xi + G y$$
$$u = H \xi + K y$$

controllore dinamico (di ordine n-p) PROPRIO

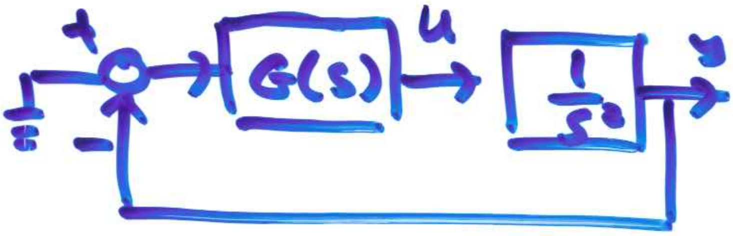
$$F = A_{22} - N A_{12} + L H_2$$
$$G = M + L (H_1 + H_2 N)$$
$$H = H_2$$
$$K = H_1 + H_2 N$$

$$G_c(s) = K + H [sI - F]^{-1} G$$

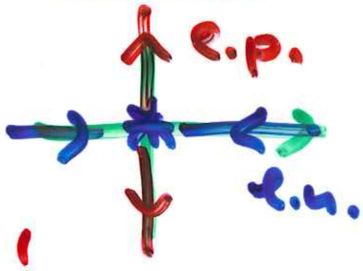
$$\sigma \begin{cases} * \; A + B [H_1 \; H_2] \\ * \; A_{22} - N A_{12} \end{cases}$$

Esempio (di CONFRONTO)

$P(s) = \frac{1}{s^2}$

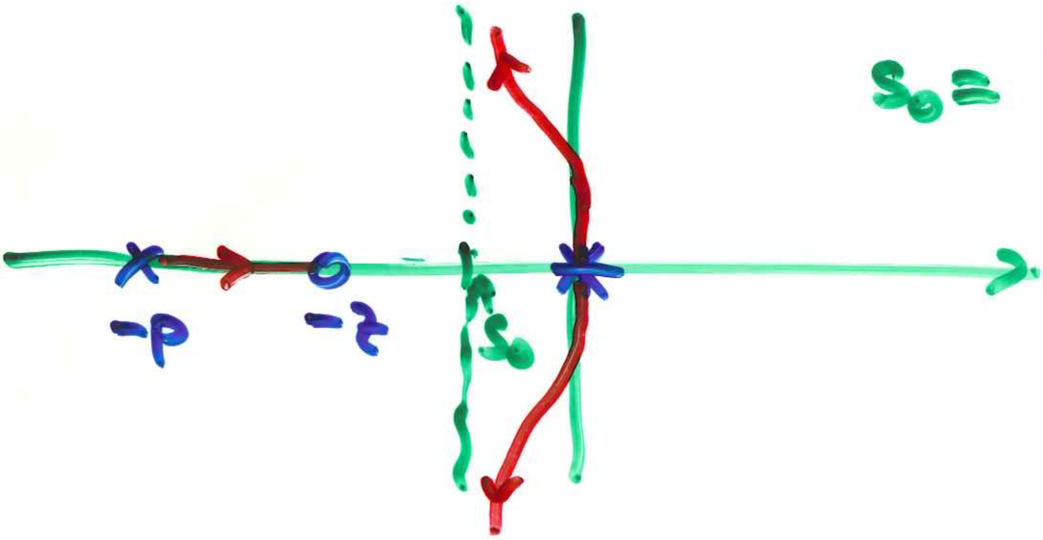


$G(s) = k$ stabilizza?



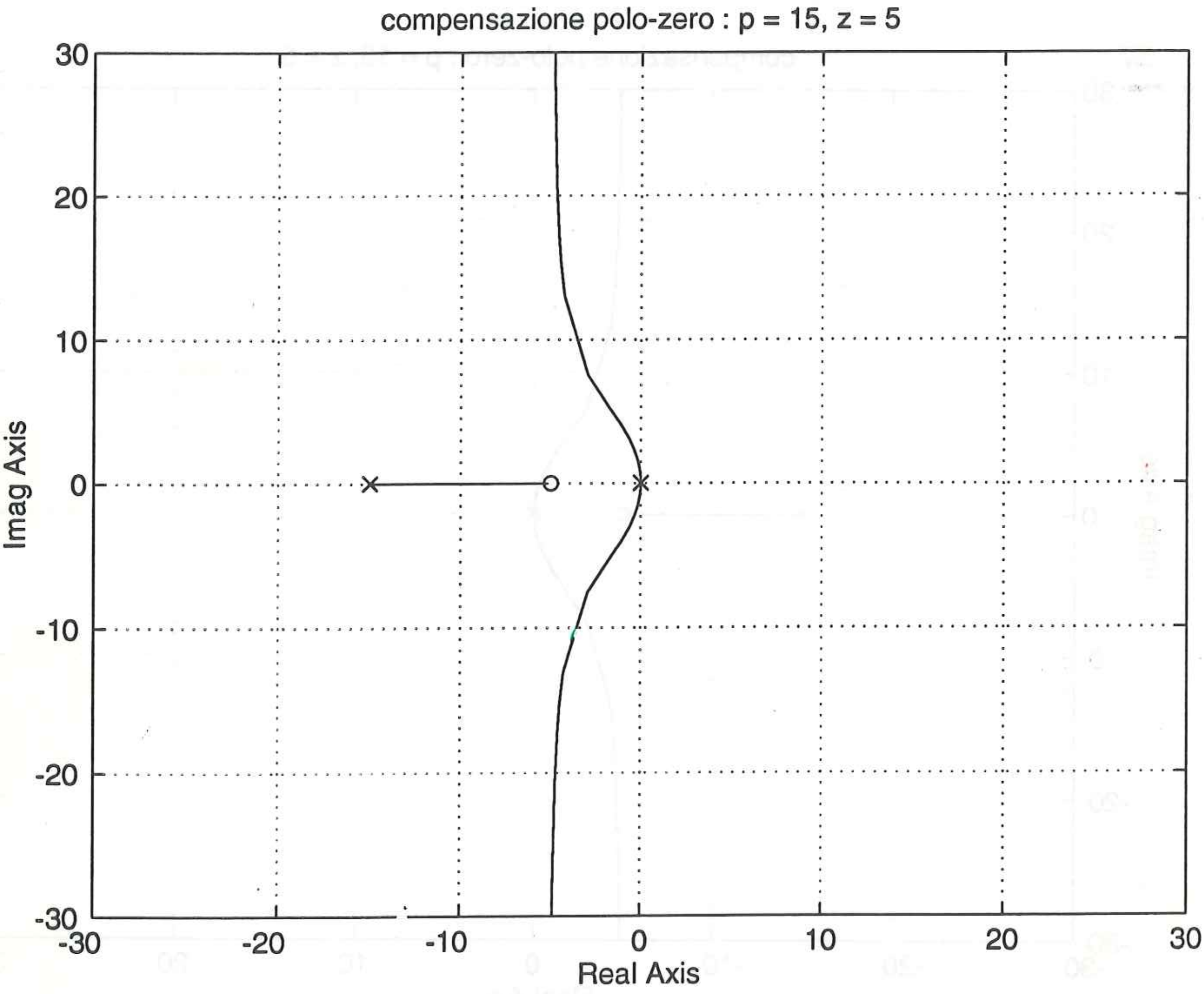
No!

$G(s) = k \frac{s+z}{s+p}$ $\quad p > z > 0 \quad k > 0$ suff. elevato



$s_0 = \frac{-p+z}{2} < 0$

$p = 15 \quad z = 5$



compensazione polo-zero : p = 15, z = 5
30
20
10
0
-10
-20
-30
Imag Axis
-30
-20
-10
0
10
20
30
Real Axis

$P(s) \rightarrow (A, B, C)$ $\qquad x = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$

$\dot{x} = Ax + Bu$

$y = Cx$

$A = \left[\begin{array}{c|c} 0 & 1 \\ \hline 0 & 0 \end{array}\right] \qquad B = \left[\begin{array}{c} 0 \\ \hline 1 \end{array}\right]$

$C = \left[\begin{array}{c|c} 1 & 0 \end{array}\right]$

I) $u = Kx$ $\qquad p^*(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2)$

$= \lambda^2 \underbrace{-(\lambda_1 + \lambda_2)}_{\alpha_1}\lambda + \underbrace{\lambda_1\lambda_2}_{\alpha_0}$

$\Rightarrow K = [-\alpha_0 \quad -\alpha_1]$

II) $\dot{\xi} = (A_{22} - NA_{12})\xi + \underbrace{L}_{B_2 - NB_1} u + \underbrace{M}_{A_{21} + A_{22}N - NA_{11} - NA_{12}N} y$

$\Downarrow$

$\dot{\xi} = -N\xi + u - N^2 y$

$w = \xi + Ny$

i) + ii)

$$u = \underbrace{\begin{bmatrix} -\alpha_0 & -\alpha_1 \end{bmatrix}}_{K} \begin{bmatrix} y \\ w \end{bmatrix}$$

x_1

stima di x_2

$$w = \xi + Ny$$

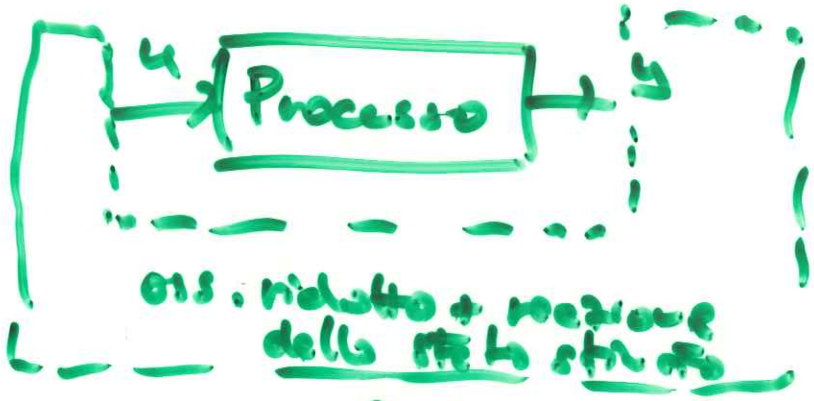
$$= -\alpha_1 \xi - (\alpha_0 + \alpha_1 N) y$$

$$\dot{\xi} = -N\xi + u - N^2 y$$

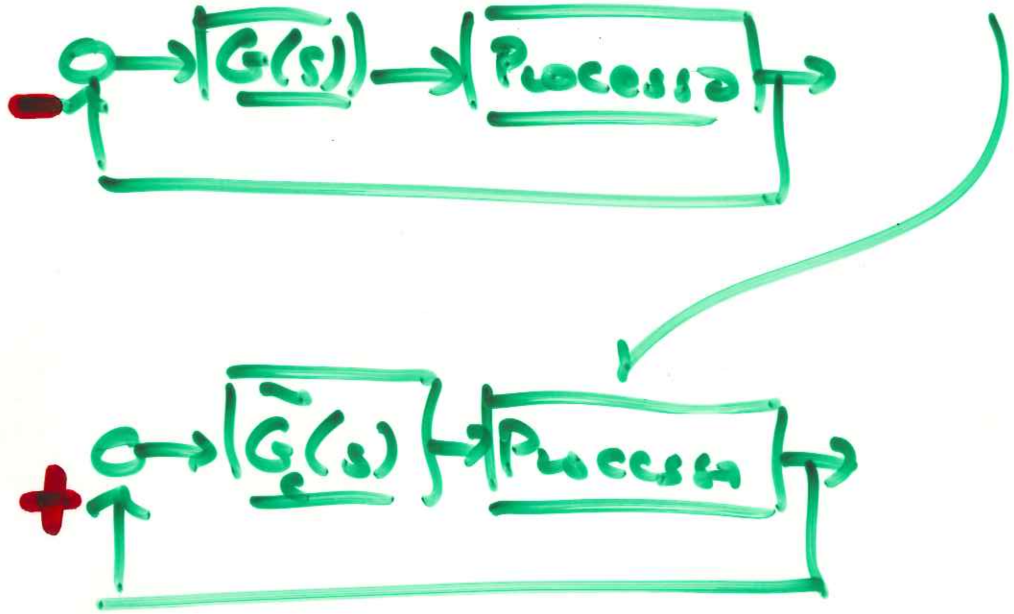
$$\dot{\xi} = -(N+\alpha_1)\xi - (\alpha_0 + \alpha_1 N + N^2) y$$

$$G_c(s) = \frac{u(s)}{y(s)} = -\frac{(\alpha_0 + \alpha_1 N)s + \alpha_0 N}{s + (\alpha_1 + N)}$$

$$= -\underbrace{(\alpha_0 + \alpha_1 N)}_{K} \frac{s + \overbrace{\frac{\alpha_0 N}{\alpha_0 + \alpha_1 N}}^{z}}{s + \underbrace{(\alpha_1 + N)}_{p}}$$



$$G_c(s) = -\tilde{G}_c(s)$$

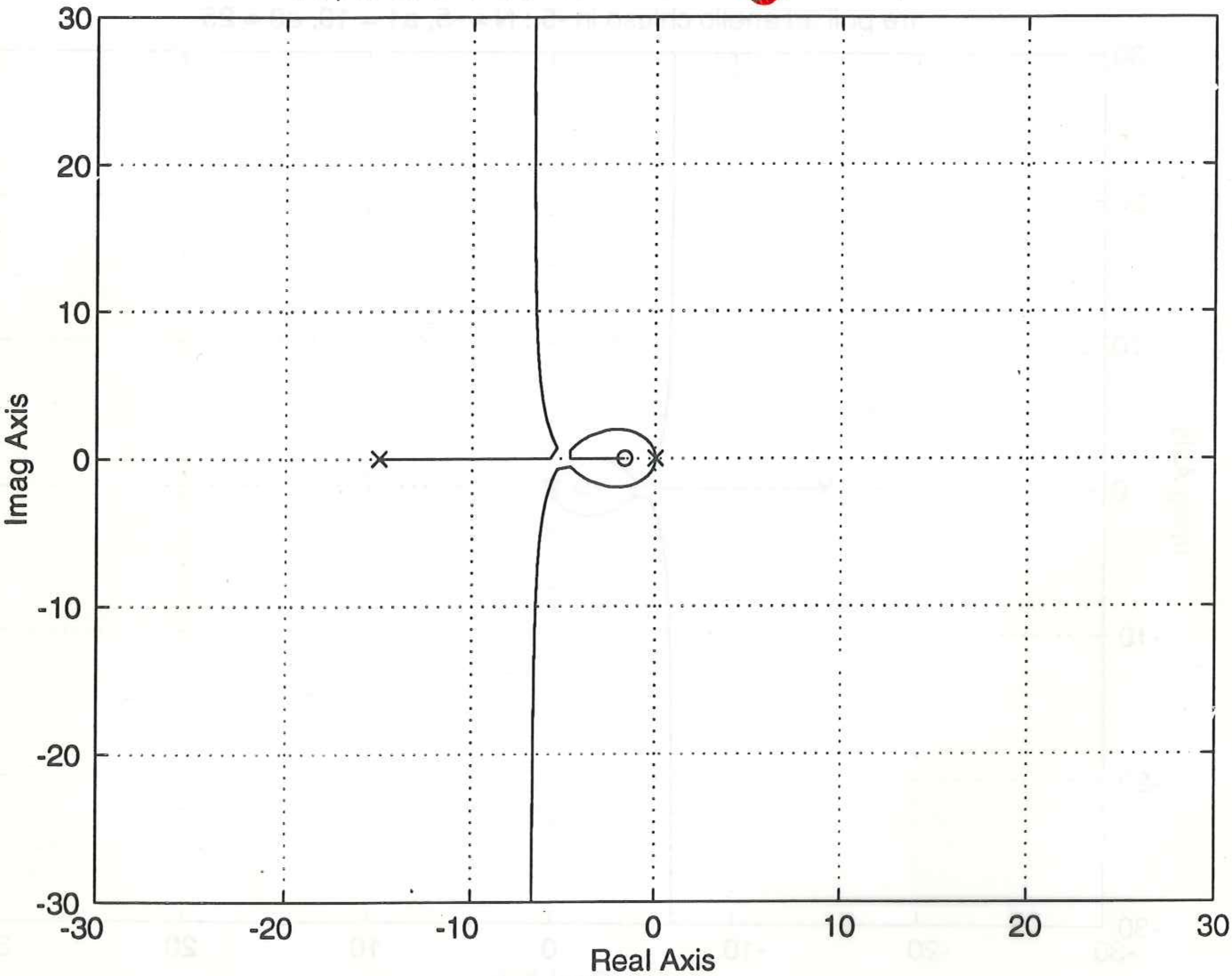


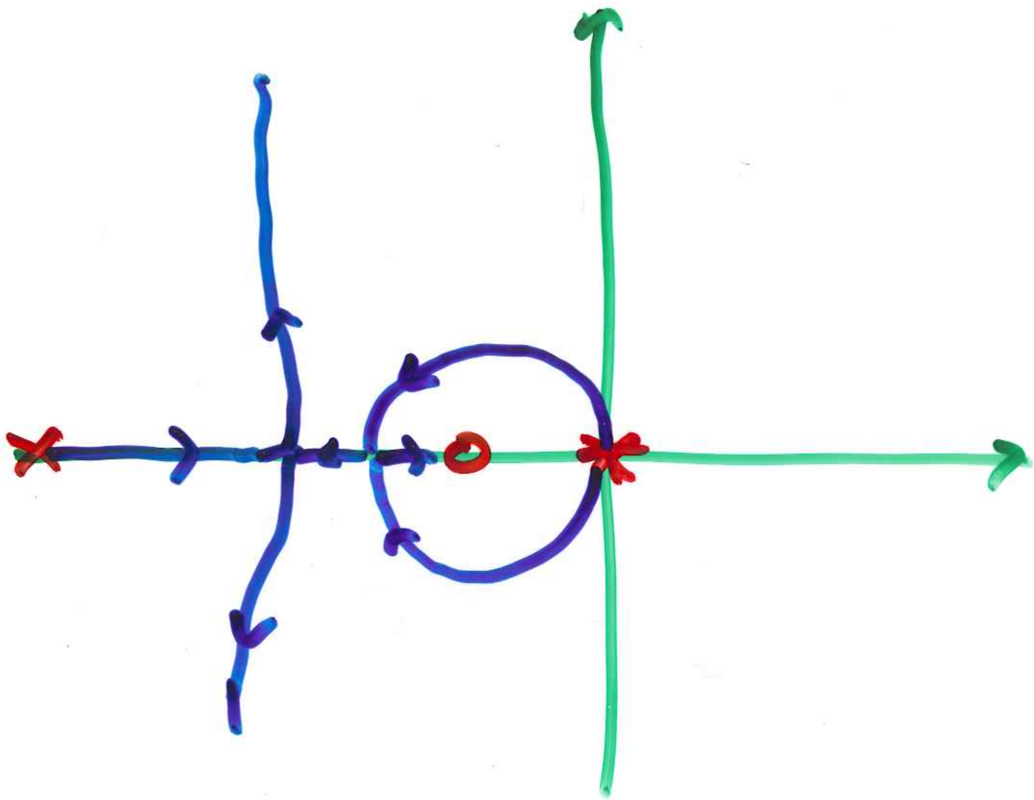
es tutti i poli ad anello chiuso in -5
(gli autovalori)

$$\lambda^2 - \underbrace{(\lambda_1 + \lambda_2)}_{\alpha_1 = 10}\lambda + \underbrace{\lambda_1\lambda_2}_{\alpha_0 = 25} \qquad \lambda_1 = \lambda_2 = -5$$

$N = 5$

tre poli ad anello chiuso in -5 : N = 6, a1 = 10, a0 = 25





$N = 15$

$$\underbrace{\lambda_1 = -5 \qquad \lambda_2 = -10}_{\alpha_0 = 50,\ \ \alpha_1 = 15}$$

$K = (\alpha_0 + \alpha_1 N) \leftarrow 275$